

In the last chapter (which was written in collaboration with A. Prástaro), the authors return to the d'Alembert equation (4), and they make a detailed study of it from the point of view of the modern geometric theory of partial differential equations. This chapter, unlike the earlier ones which require only a background subsumed in the usual graduate course in real analysis, requires more mathematical sophistication. Terms like tensor products, n -dimensional manifolds, tangent bundles, cotangent bundles, the tangent space at a point, fibre bundles, etc., appear in Chapter 8 for the first time. The reader not versed in differential geometry may find the going a little rough here.

The book concludes with a section stating seven open problems.

I found this to be a well-written book with a unified approach to a subject whose main results had heretofore only been located in journal articles and unpublished manuscripts. I can heartily recommend it to anyone who is interested in knowing when a function of many variables can be represented as the sum of a product of functions of single variables.

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Ahmed I. Zayed, *Handbook of Function and Generalized Function Transformations*, CRC Press, Boca Raton, FL, 1996, xxii + 643 pp.

This volume completes a collection of books devoted to various classes of one- and two-dimensional integral transformations which are used as important mathematical tools for solving problems in all areas of physical sciences.

As is known in the theory of integral transformations, the Laplace, Fourier, and Mellin transforms play an important role, not only in solving many problems in applications, but also in the composition structure and mapping properties of other integral transforms. This means that many integral transformations were constructed as a composition of the classical transforms mentioned above and some substitution operator.

The author gives useful information about the theory and applications of these transforms based on classical books of R. Churchill, I. Sneddon, E. C. Titchmarsh, and A. Zemanian. This includes mapping properties in the classical function spaces as well as in spaces of generalized functions, existence theorems, inversion formulas, the Parseval relation, convolution properties, and applications in finding classical and generalized solutions of different classes of differential and integral equations.

The author made this book self-contained and included all preliminary material from real, complex, and functional analysis, generalized functions and Schwartz distributions, theory of special functions and orthogonal polynomials. This material is contained in Chapters 1–4. Further parts consist of a collection of linear and non-linear (the Zak transform) transformations. This includes the category of integral transformations known as *Mellin convolution type transforms*: the Hankel, Stieltjes, Hilbert, Meijer, Hartley, Mittag-Leffler, Weierstrass, Abel, Y -, I -transforms, the Riemann–Liouville and Weyl fractional integrals, the hypergeometric transforms, the G -, H -, and E -transforms. The essential part of the book is devoted to *the Fourier-type transforms*, the discrete transforms, the wavelet transforms, and the Radon transforms and to the important class of the integral transformations that depend upon the parameter or index of a hypergeometric function in the kernel (*index transforms*) such as the Kontorovich–Lebedev transform, the Mehler–Fock transform, and the index ${}_2F_1$ -transform.

It should be noted that over the past 30 years the theory of integral transformations has been intensively developed and various methods and approaches of obtaining new transforms and their inversions have been discovered, which has resulted in increased investigation of the new properties for known integral transformations. We mention for example the composition

method by N. Lebedev, J. Wimp, S. Yakubovich, Vu Kim Tuan, H.-J. Glaeske, and others; the hypergeometric approach based on the Mellin transforms of hypergeometric functions by O. Marichev; the convolution method by I. Dimovski, H.-J. Glaeske, S. Yakubovich, and V. Kacichev; and the operational method by V. Ditkin and A. Prudnikov.

As a conclusion, we invite all users of integral transformations and their applications to read this book. The references of this volume could be completed by mentioning the following recent monographs on integral transformations and related topics: *Handbook of Integral Transforms of Higher Transcendental Functions*, by O. Marichev (1983); *The Double Mellin–Barnes Type Integrals and Their Applications to Convolution Theory*, by Nguyen Thanh Hai and S. Yakubovich (1992); *The Hypergeometric Approach to Integral Transforms and Convolutions*, by S. Yakubovich and Yu. Luchko (1994); and *Index Transforms*, by S. Yakubovich (1996).

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Levan Zhizhiashvili, *Trigonometric Fourier Series and Their Conjugates*, Mathematics and Its Applications **372**, Kluwer, Dordrecht, 1996, xii + 300 pp.

Fourier series of one variable have been studied since Fourier (1769–1830) and the results have been relevant in various areas of mathematics, but also in the applied sciences, in particular physics, signal processing, and mechanics. Quite a few books deal with Fourier series of one variable, both from the theoretical and the practical point of view. In contrast with this, hardly any book is known that treats multiple trigonometric series, i.e., Fourier series of several variables. The present book partly fills this gap.

The book was first published in 1993 in Tbilisi, Georgia. This translation will surely make the book more easily available. There are two parts: part 1 deals with simple trigonometric series, and in part 2 multiple trigonometric series are covered. There are four chapters in part 1, in which the familiar results for the classical Fourier series of a function f of one variable are discussed. Right from the beginning, much emphasis is given on the conjugate function \tilde{f} and the Hilbert transform \tilde{f} .

Chapter 1 starts with the L^p -spaces and the H^p -spaces, moduli of smoothness, Privalov's theorem regarding Fourier series and their conjugates for Lipschitz functions and various extensions. Kolmogorov's theorem (that $\|\tilde{f}\|_p \leq A(p) \|f\|_1$ for $0 < p < 1$), Titchmarsh's theorem

$$\int_{-\infty}^{\infty} \frac{|\tilde{f}(x)|^p}{1+x^2} dx < \infty, \quad 0 < p < 1$$

for $f \in L(\mathbb{R})$, and Kober's improvement of Titchmarsh's theorem are treated in detail. A result by Hardy and Littlewood that $f \in \text{Lip}(\alpha, p)$ for either $1 < p < \infty$ and $0 < \alpha \leq 1$, or $p = 1$ and $0 < \alpha < 1$ implies that also $\tilde{f} \in \text{Lip}(\alpha, p)$, gets detailed attention, and some improvements are given.

Chapter 2 is about pointwise convergence and summability of trigonometric series. Various classical results on convergence and divergence of trigonometric series are presented. The Cesàro means of the partial sums of the Fourier series and the partial sums of the conjugate series are considered here in detail. Chapter 3 deals with convergence and summability in the spaces $L^p([-\pi, \pi])$, for $0 < p \leq \infty$. The cases when $0 < p \leq 1$ and $1 \leq p \leq \infty$ are treated separately, and Fourier series (and their conjugate series) for odd and even functions are singled out. Chapter 4 finally gives some approximating properties of the Cesàro means.